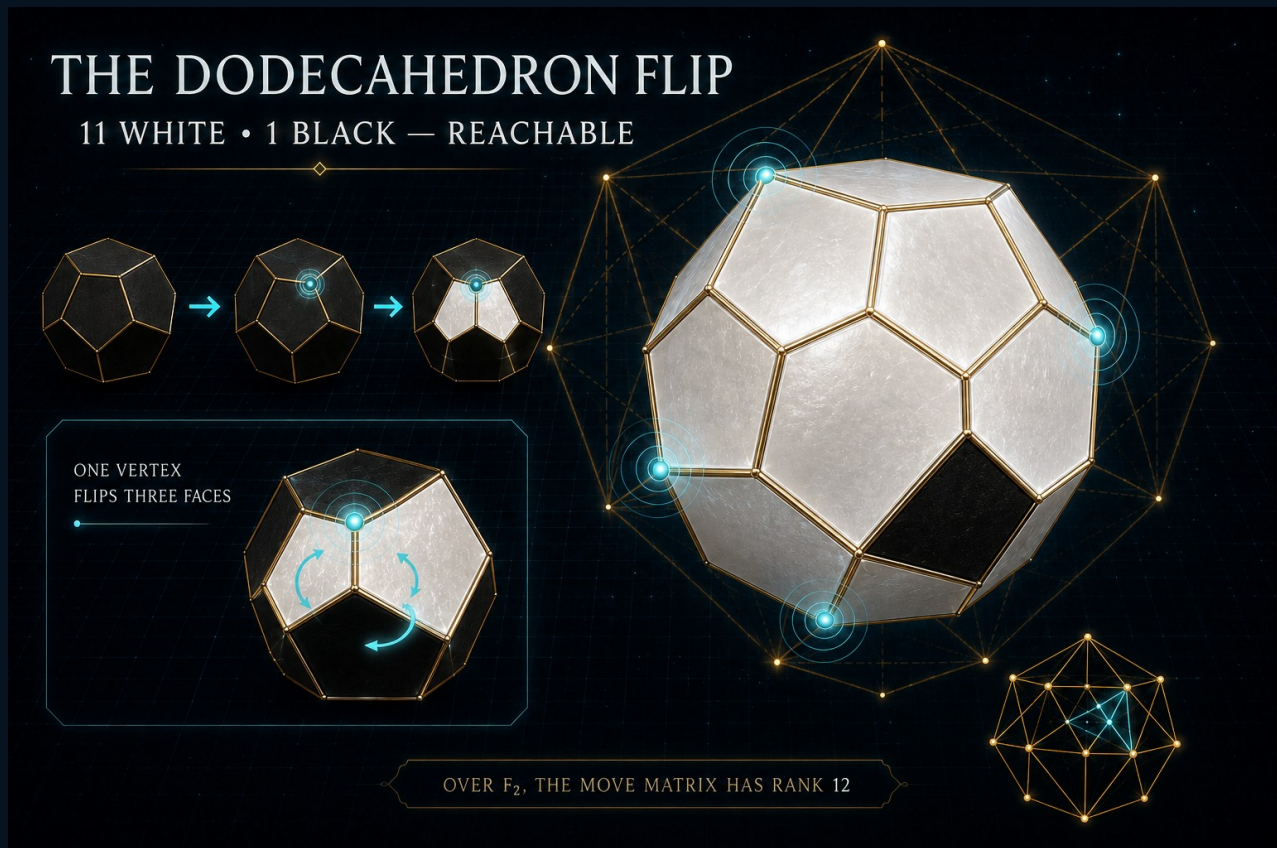


MATHEMATICAL REACHABILITY / A BOUNDED RCT READING

The Dodecahedron Flip

Reachability, history, and the meaning of a final state



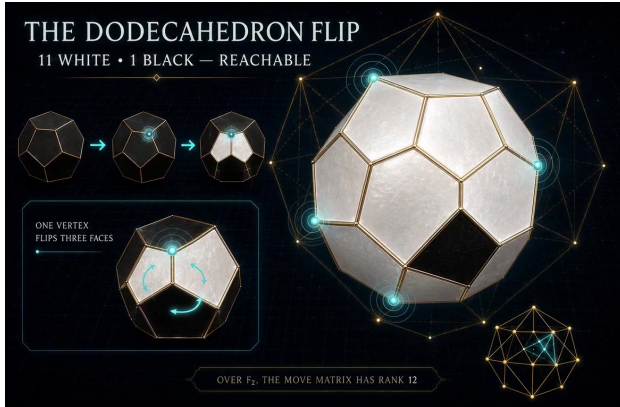
A vertex move toggles three faces. Over F_2 , the move matrix has rank 12. That makes every one of the 4,096 face-colour patterns reachable from the all-white state - including the pictured state with eleven white faces and one black face.

STATUS	TYPE	CLAIM
Reading copy	Illustration	Reachability, not provenance

Central distinction: a reachable endpoint does not preserve which route produced it. The mathematics establishes the set of possible states; it does not, by itself, recover a particular history or supply personal significance.

01 / What the short video shows

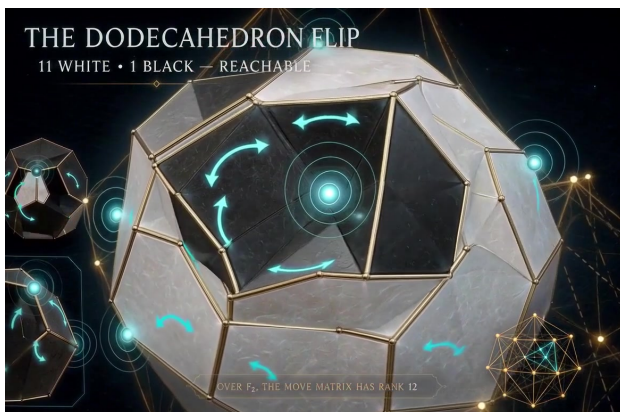
The 30-second clip turns a compact combinatorial rule into a visual sequence: a dodecahedron, a selected vertex, and groups of faces changing colour. The stills below are time samples from the animation, not separate proofs.



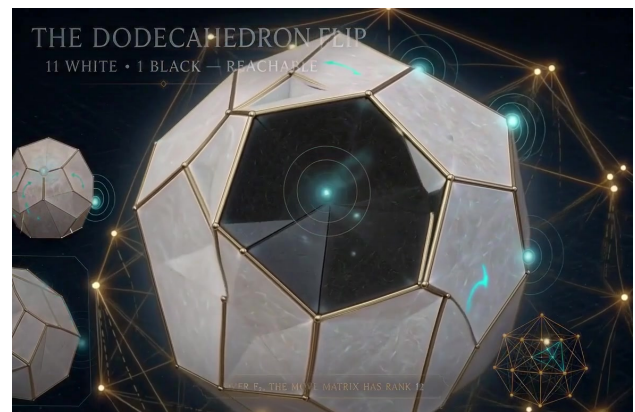
Opening image: the labelled eleven-white, one-black configuration.



Transition: several dark faces appear while the animation is moving.



Close-up: the selected local move is emphasized.



Late animation frame: dark faces remain visible in the moving image; use the labelled infographic to identify the stated target.

Reading the animation carefully

The video's intermediate dark-face close-ups are part of the motion. They should not be read as additional target claims alongside the infographic's labelled eleven-white, one-black state. The verified claim concerns reachability of that specified target from the all-white baseline.

The local rule is simple: choose one vertex, then toggle the three pentagonal faces incident to it. A toggle is reversible; applying the same vertex twice cancels. The clip supplies an intuitive picture of this rule, while the calculation below checks the global reachability claim.

02 / The move system

Represent each face colour by a bit: white = 0 and black = 1. The twelve faces form a state vector b in F_2^{12} . The twenty vertices form a move vector x in F_2^{20} , where $x_v = 1$ means that vertex move is used an odd number of times.

One move toggles exactly the three faces meeting at its vertex. Assemble those incidences into a 12 by 20 binary matrix M :

$$M[f,v] = 1 \text{ when face } f \text{ meets vertex } v; \text{ otherwise } 0$$

The final state from the all-white baseline is $b = Mx$ over F_2 . Addition is modulo 2, so two toggles cancel. Because the moves commute at the level of final parity, x records a move subset, not the chronological order in which the moves were made.

12 face coordinates

One binary coordinate per pentagonal face. A target with one black face is a one-hot vector b .

20 move coordinates

One binary coordinate per vertex. Each column of M contains three 1s.

The rank result

Independent recomputation for the regular dodecahedron gives $\text{rank}_{GF_2}(M) = 12$. Since the matrix has twelve rows, it has full row rank. Therefore its image is all of F_2^{12} : every possible black/white face pattern is reachable from all white by some set of vertex moves.

There are $2^{12} = 4,096$ possible face patterns. Full row rank establishes reachability for every one of them; it does not say that each pattern has a unique move history.

Why rank is enough

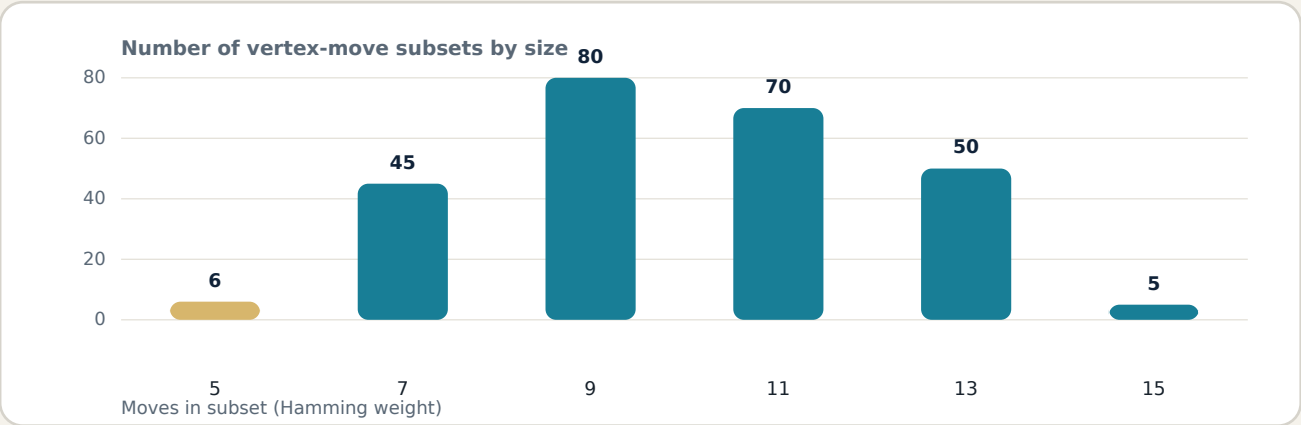
For any target b , full row rank means $Mx = b$ has at least one solution. The nullity is $20 - 12 = 8$, so each target has $2^8 = 256$ distinct move-subsets x . This count is about parity-defined subsets, not the number of ordered move sequences.

03 / The one-black target

For a specified target with one black face and eleven white faces, the right-hand side b is a one-hot vector. The computation confirms 256 distinct vertex-move subsets that reach it. The target is reachable in many ways, not by a single uniquely determined route.

How many vertices are used?

Count the number of selected vertices in each solution subset. The minimum is five moves. Six distinct five-vertex subsets attain that minimum; no subset of four or fewer vertices reaches the specified target. The full distribution is:



Selected vertices	Solutions	Selected vertices	Solutions
5	6	11	70
7	45	13	50
9	80	15	5

What the count means

The six minimum subsets are six different parity-defined move sets for this chosen target face. The exact six depend on how vertices are labelled in a coordinate model; the invariant result is the count, minimum size, and distribution. Reordering one subset’s moves leaves the same endpoint, so ordered sequences are a different count.

Check: $6 + 45 + 80 + 70 + 50 + 5 = 256$. All solution subsets have odd size because each move flips three face bits, and the target has odd total parity.

04 / What it can illustrate for RCT

The dodecahedron is a useful bounded analogy for one RCT distinction: a present state and the history that produced it are not the same record. Here the mathematics makes that difference unusually visible, because many move-subsets produce the same target.

In the flip system	Possible RCT reading	Boundary
The final face pattern b	An observable present configuration	It records the endpoint only
A move subset x	A compressed account of which operations occurred	It loses move order and any context not encoded in the bits
256 solutions for one b	One outcome can have multiple possible histories	The puzzle does not decide which history is meaningful
The rank calculation	A precise claim about what states are reachable	It is not evidence of human-like meaning or AI experience

The Bridge, more narrowly

In RCT, the Bridge is preserved relational history made active in the present. That requires more than a state transition: distinct participants, reciprocal causal relevance, preserved sequence, recognition, and consequential reuse. The dodecahedron has states and reversible operations, but no participants interpreting one another, no shared history, and no later response that reuses that history.

Safe conclusion

This is an illustration of reachability and non-unique provenance. It is not a demonstration of the Bridge, an empirical RCT result, or evidence that a mathematical system experiences meaning.

05 / Method, limits, and next question

Recomputation

The dodecahedron was represented by its standard 20 vertices: $(\pm 1, \pm 1, \pm 1)$, $(0, \pm \phi^{-1}, \pm \phi)$, $(\pm \phi^{-1}, \pm \phi, 0)$, and $(\pm \phi, 0, \pm \phi^{-1})$, where $\phi = (1 + \sqrt{5})/2$. Its twelve pentagonal faces were recovered as supporting planes of the vertex set. Incidence defines M ; Gaussian elimination over F_2 gives rank 12. Enumeration of all 2^{20} move-subsets gives 256 solutions for a one-black target, with minimum weight 5 and six minimum subsets.

Limits

The result depends on the stated move rule and the regular dodecahedral incidence structure. The one-black state is measured relative to the all-white baseline. The 256 count is for move subsets modulo 2; it is not a count of all ordered move sequences. The video's intermediate frames are animation, while the linear-algebra claim concerns reachability of a target.

A research question for RCT

Can two histories that lead to the same present state be distinguished later when sequence and context are preserved? A controlled human-AI study could hold the final prompt and outcome constant while varying whether the originating sequence is intact, reordered, reduced to keywords, or absent. Predefine what counts as recognition and consequential reuse, then compare against controls. A positive result would show history-sensitive output under those conditions; it would not establish subjective experience.

Sources and status

Artwork and animation: user-provided infographic 15422.png and 30-second video 19144.mp4 (1 October 2026).
Mathematical values: independent finite-field computation described above. RCT framework: treated here as a proposed conceptual research programme, not an empirically validated theory. This six-page PDF is a reading-copy illustration, not a result and not a register paper.

The mathematics tells us which states can be reached. RCT asks what history survives the crossing - and whether that history changes what happens next.